POSSIBILISTIC TIME SERIES

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Abstract A time series (or a set of it) reveals how a state variable of a system behaves with time. In this article, we propose a method and a tool, as well, for showing how possibilistic information (fuzzy sets) helps generate time series. This might open a possibility of viewing a system as an entity governed by some linguistic belief rather than an entity governed by some analytical equations.

Keywords: *Orbit; Time Series; Fuzzy Sets; State Variable*

INTRODUCTION

When we study systems, specially the nonlinear dynamical systems, we identify some state variables, and see how their values vary with time. A set of such values is called time series. Time series is thus a format of information from which varies decisions are made regarding different aspects of a system (we are talking about).

From mathematical point of view, a time series (i.e., the system) is an entity called *orbit*, which starts from a starting point known as *seed*. The seed is then input in the system of *state equation*(s), which determines the value of the state variable for the next time period. This iteration goes on until a definite period of time or until the value of the state variable converges to a single value. When the state equation is $X^2 + c$ (X and c are real numbers), we write the orbit f_c, as follows:

$$f_c : \mathbf{R} \to \mathbf{R}, \quad f_c : X \mapsto X^2 + c \quad (X, c \in \mathbf{R}) \quad (1)$$

For example, if seed is 0.8 and $f_{0.1}$ is as follows,

0.8 (seed) → 0.7400 → 0.6476 → 0.5194 → 0.3698 → 0.2367 → 0.1560 → 0.1243 → 0.1155 → 0.1133 → 0.1128 → 0.1127 → 0.1127 → 0.1127 → 0.1127,….

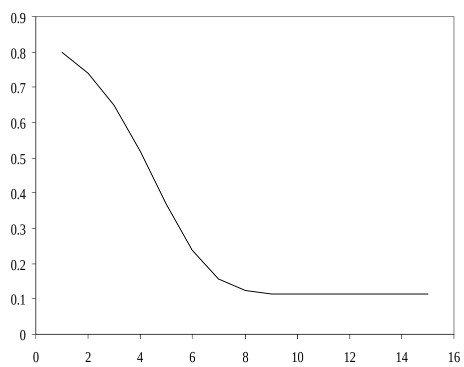

Fig. 1 An orbit for f_c

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Figure 1 illustrates this orbit. What is orbit says is that when we start from 0.8 the state variable stabilizes at the value of 0.1127 following a path of continuos decrease, parabolically.

Changing the values of seed and c in (1) we can generate a family of time series that represents the nature of many nonlinear dynamical systems we encounter [Lyubich, 2000] in various engineering fields. Another widely used state equation in generating orbits is $X(t + 1) = aX(t)(1 – X(t))$, which is know as logistic equation.

However, our argument here is that if we use fuzzy sets in generating time series we could conclude that some possibilistic rules actually controls the behavior of the system. This could lead to the fact that a system is an entity governed by some linguistic belief rather than an entity governed by some analytical equations.

See references [Zadeh, 1978, 2001] for mathematical meaning of possibility, how effective it is in encoding the humanistic/linguistic information, which is the primary source of information from analysis or synthesis. This in turn makes the whole system analysis more computer- and human-friendly.

METHOD

Consider that we have a finite number of fuzzy sets $F = \{Fi \mid i = 1,\ldots,n\}$ in a universe of discourse $U = \mathbf{R}$. The membership function of *i*-th fuzzy set is $f_{Fi}(u) \in [0, 1]$, $u \in U$ (=**R**). For convenience, we refer to a value in U as u-value, which is for this case a real number. Since how much a fuzzy set influence a u-value in terms of degree of belief is the membership value, the quantity $u \times f_{Fi}(u)$ quantifies the possibilistic expectation on that value (u-value). As such, if we add all these expectations together we get a quantity called *expected fuzzy value* of a u-value in that particular system of fuzzy sets F

[Ullah, 2000]. The expression of expected fuzzy value ($F_{evF}(u)$) is as follows,

$$F_{evF}(u)=u\sum_{i=1}^{n} f_{Fi}(u) \qquad (2)$$

We can use $F_{evF}(u)$ as a means of iteration in generating an orbit that we should call *fuzzy orbit* in the sense that it has been generated by a set of fuzzy sets. The fuzzy sets used then represent the (possibilistic) governing factors of u-values (i.e., the values of a state variable with respect to time). In mathematical notation, the fuzzy orbit FO is as follows,

$$FO: U \to U, \quad FO: u \mapsto F_{evF}(u) \quad (u \in U = \mathbf{R}) \qquad (3)$$

TOOL

We developed an application called "*Fuzzy Orbit*" for generating fuzzy orbits using three fuzzy sets with triangular fuzzy number. This application also determines also other informational characteristics of a system of fuzzy sets, as well [Ullah, 2000]. One of the user interfaces of the application is shown in Fig. 2. It also displays the time series using the numerical values of the fuzzy orbit according to (3), as shown in Fig. 3. At the same times, it stores these values in Excel files for reporting and transferring to other locations using networks (e.g., Internet).

Figure 2 shows a case of three fuzzy sets with triangular fuzzy numbers (0, 20, 40), (13, 26, 60), and (18, 35, 70). The seed is 28. This produces an expected fuzzy value of 59.6235, which again produces another expected fuzzy value of 18.3368 according to (3). This way the fuzzy orbit proceeds until 25 iterations until the expected value becomes zero. Figure 3 shows the orbit generated in graphical format, produced by Fuzzy Orbit.

The time series generated by the fuzzy orbit might represent the behavior of a state variable of a system with time. If we start from a different point (i.e., we change the seed), we might get another pattern of the time series. Again, if we change the fuzzy sets, we might get a different time series.

APPLICATION

We can apply the concept of fuzzy orbit in getting a time series similar to that we get for a state variable in reality. One of the applications we find in [Ullah, 2001]. However, here we try to find out the serious points of the applications in general, as follows. We can use a single orbit in order to generate the time series. Alternatively, we can use several orbits connected piecewise one after another. For the second option, it is expected that the orbits should be the nearby orbits (i.e., the difference among the seeds should not be large.) This expectation will also help in explaining the phenomenon related to the state variable using other tools in analyzing nonlinear dynamical systems, especially using chaos theoretic tools.

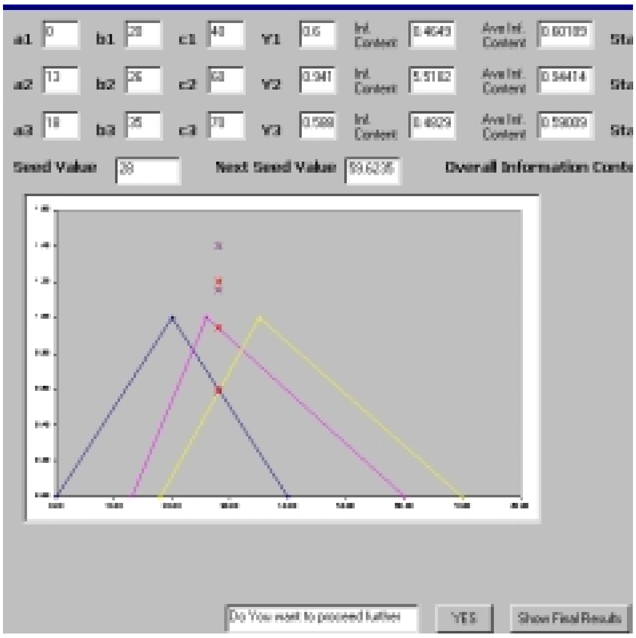


Fig. 2 One of the user interface of "Fuzzy Orbit"

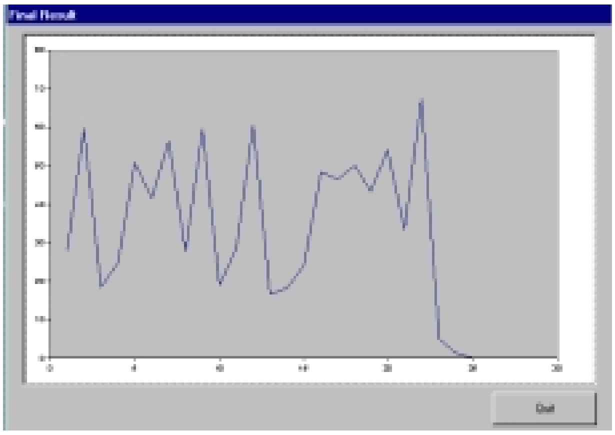
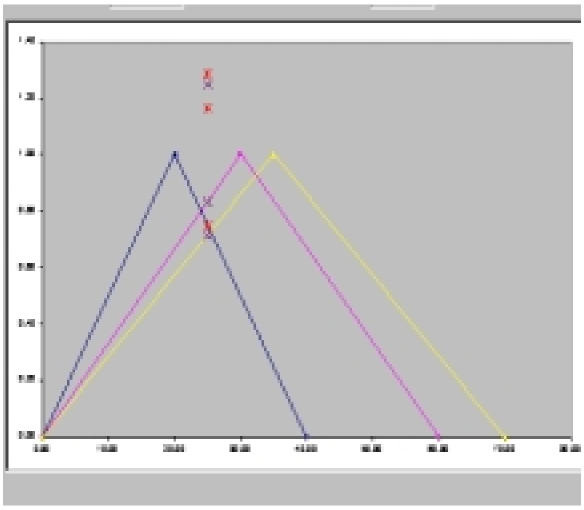
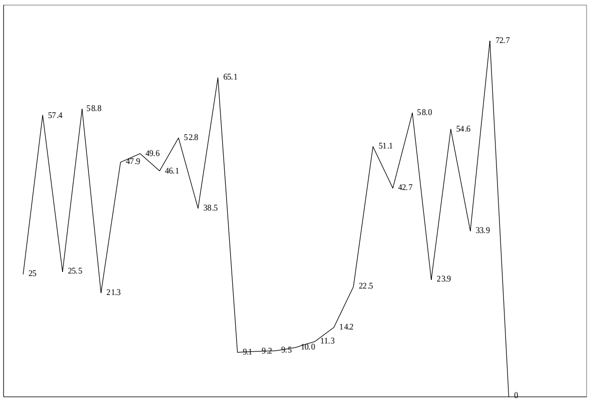


Fig. 3 A plot of fuzzy orbit

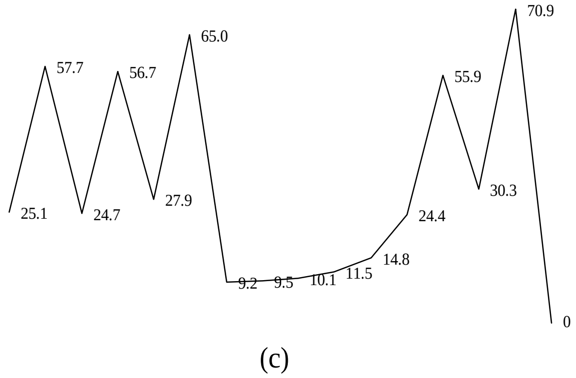
Figure 4 shows a system of fuzzy sets and two orbits generated from seeds 25 and 25.1, respectively. There are certain level of noise and periodicity in these orbits as evident from Fig. 4. If we connect these orbits we could generate a time series of a hypothetical state variable that shows periodicity and noise, and the periodicity along with the noise have some variation with time. One of the possibilities is shown in Fig. 5, which contains five nearby orbits in the order of orbit of seed 25 $\to$ orbit of seed 25.1 $\to$ orbit of seed 25 $\to$ orbit of seed 25 $\to$ orbit of seed 25.1. We make the connection between orbits by omitting the zero values. This means that when the orbit of seed 25 was connected with orbit of seed 25.1, the connection was made between 72.7 (the value just before zero) and 25.1.



(a)



(b)



(c)

Fig. 4 A system of fuzzy set and two orbits with seeds 25 and 25.1, respectively

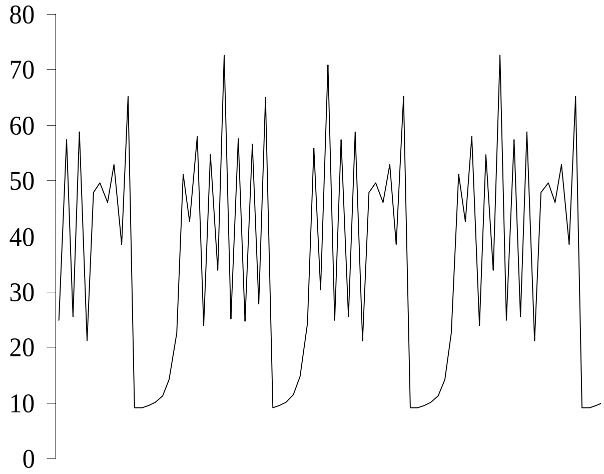


Fig. 5 A time series of piecewise fuzzy orbits

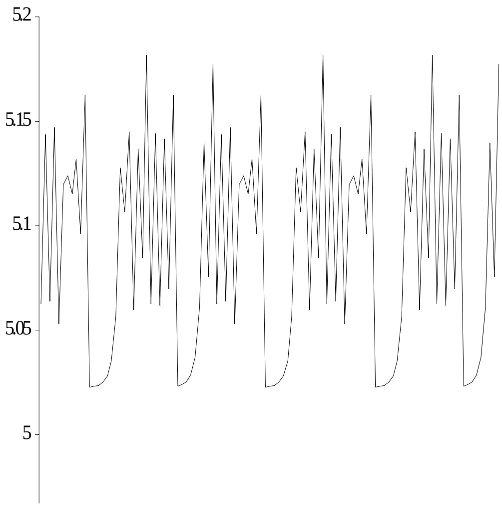


Fig. 6 Time series of Fig. 5 with magnitude readjusted

We can use other functions in order to adjust the magnitude to a different scale with out destroying the nature of the time series. For example, if we use a sigmoid function 10/(1+exp(-0.001X), where X is the value of the state variable in the time series, we get another time series, as shown in Fig. 6. Here, due to the use of this function the magnitude scale of the state variable have readjusted to 5.0—5.2 from 0—10, and the nature of the time series remains the same. However, it is also possible to eliminate the noise, choosing suitable values of the coefficients in the sigmoid.

CONCLUSION

We proposed a method for generating time series of a state variable of nonlinear dynamical systems using a concept called fuzzy orbit, which is an orbit that uses the expected fuzzy value of a value in the universe of discourse. We use an application called "Fuzzy Orbit" as a tool for generating fuzzy orbit from a system of fuzzy sets with three triangular fuzzy numbers. We show how this tool helps us in getting a time series with noise and periodicity. Further study is needed in order investigate the theoretical and application aspects of the method in details.

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